24-9-2015	Std. XII	MATHEMATICS	Time : 3 hrs.	Marks : 100

General Instructions :

(i) **All** questions are compulsory.
(ii) Questions 1 to 6 is Section A are Very Short Types carrying 1 markes each.
(iii) Questions 7 to 19 in Section B are Long answer I Type questions carrying 4 marks each.
(iv) Questions 20 to 26 in Section C are Long Answer II Type questions carrying 6 marks each.
(v) Please write down the serial number of the question before attempting it.

SECTION-A

1. Write a 3×3 skew symmetric matrix.

2. If the binary operation * on the set Z of integers is defined by $a * b = a + b - 5$ then write the identity element for the operation * on Z.

3. What is the range of the function $f(x) = \dfrac{|x-1|}{x-1}$?

4. If $\int\left(\dfrac{x-1}{x^2}\right) e^x\,dx = f(x).\, e^x + c$, then write the value of f(x).

5. Write the principal value of $\tan^{-1}\left(\sqrt{3}\right) - \cot^{-1}\left(-\sqrt{3}\right)$

6. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n, find the value of Det (A^n)

SECTION-B

7. Let $f(x) = x - |x - x^2|, x \in [-1, 1]$. Find the point of discontinuity, (if any), of this functions on $[-1, 1]$.

8. If $y = x^3 \log\left(\dfrac{1}{x}\right)$ then prove that $x\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 3x^2 = 0$

OR

Verify mean value theorem for the function $f(x) = (x-4)(x-6)(x-8)$ on the interval $[4, 10]$

9. If $\dfrac{x}{x-y} = \log\left(\dfrac{a}{x-y}\right)$ then prove that $\dfrac{dy}{dx} = 2 - \dfrac{x}{y}$

10. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \dfrac{\pi}{2}, x, y, z > 0$ then find the value of $xy + yz + zx$

11. Check whether the relation R in R defined by
$R = \{(a, b) : a \le b^2\}$ is reflexive, symmetric or transitive.

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Let $f: N \longrightarrow N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{if } n \text{ is odd} \\ \frac{n}{2}, \text{if } n \text{ is even} \end{cases}$$

for all $n \in N$. Find whether the function f is bijective.

12. If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$, $(\theta \neq 0)$, then find the value of θ

OR

If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \ldots\ldots + \tan^{-1}\left[\frac{1}{1+n(n+1)}\right] = \tan^{-1}\theta$, then find the value of θ.

13. Using elementary transformation, find the inverse of the following matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

14. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

15. Using differential, find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$

OR

Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \pi/4$

16. Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$

OR

Evaluate $\int \frac{\sin(x-a)}{\sin(x+a)}\,dx$

17. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

18. Find the intervals in which the function f given by
$f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$
is strictly increasing or strictly decreasing.

19. Find : $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}\,dx$

SECTION-C

20. Let A = Q × Q, where Q in the set of all rational numbers, and * be a binary operations defined on A by (a, b) * (c, d) = (ac, b + ad) for all (a, b), (c, d) ∈ A
Find
(i) the identity elements in A
(ii) the invertible element of A

OR

Let $f: N \to R$, be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$, where S is the range of f is invertible Also, find the inverse of f.

21. Find the minimum value of $(ax + by)$, where $xy = c^2$

OR

Find the coordinates of a point of the parabola $y = x^2 + 7x + 2$.
Which is closest to the straight line $y = 3x - 3$.

22. Find $\int \frac{\log x}{(x+1)^2} dx$

23. If $f(x) = \sqrt{x^2+1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f^1[h^1\{g^1(x)\}]$

24. If a, b and c are all positive and distinct, show that

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$ has a negative value

25. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to its 3, 2 and 1 students with a total award money of Rs 1000. School Q wants to spend Rs 1500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is Rs 600, using matrices find the award money for each value.
Apart from the above three values, suggest one more value for awards.

26. The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not enceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less then 3400 and 680 respectively. A man receives Rs 225 a day and a woman receives Rs 200. How many men and women helpers should be hired to keep the pay-roll at a minimum? Formulate an LPP and solve it graphically.

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